Modal Logic: Theoretical Stuff

* What is modal logic:
* Non-classical logic: classical logic is restricted to extensional construction and truth functionality. Non-classical logic incorporates richer meanings of natural language. In the case of modal logic, the value of a formula depends on the interpretation of the operator and the contexts in which the evaluation takes place.
* Intensional logic: classical first-order logic is extensional. Extension is definition by designation, intension is definition by meaning. Intensional logic deals with the intentional contexts such as “it is necessarily so..”, “it is known that…” etc
* Propositional logic is based on truth functional operators. “truth functional” means that there is a single truth value determined by such operators. Truth functional operators can be expressed with truth tables. Something is truth functional when the truth value solely relies on the terms in the formula.
* Something is non-truth functional when the truth value of a formula depends on things other than the given terms.
* Modal logic accounts for the relativity of truth values depending on the context, or in this case, possible worlds.
* Ontology: propositional logic + modal operators
* operators “□ = it is necessary that”, “◊ = it is possible that”
* Possible worlds: other situations/contexts in which the same proposition can have different values
* Leibniz first proposes the idea of “all the possible worlds which God created” and how truths are either true only in our world or true in all the worlds. Kripke reintroduces the concept as “a set of propositions that describe our world to account for different possibilities”
* Possible world semantics is the idea that the truth of the necessity/possibility of a formula depends on the truth of the formula in other possible worlds.

Definition 1: Kripke Models

* The semantics of non-classical logic (\*the notion of possible worlds)
* The basis for modal logic
* Kripke Model is a model M which consists of:

1. A nonempty set K of contexts
2. A binary relation R on K, the accessibility relation
3. A valuation function V which assigns a truth value Vk(p) to every propositional letter p in each context k K

Definition 2

* The elementary idea for modal logic
* A model M for modal propositional logic:

1. A nonempty set W of possible worlds
2. A binary relatin R on W, the accessibility relation
3. A valuation function V which assigns a truth value Vw(p) to every proposition letter p in each world w W

* Model M = Frame (W & R) + V (valuation function)
* The innovation of intensional propositional logic: truth values are relative to the possible worlds in which evaluation takes place & may depend on truth values in other such possible worlds

Definition 3

* Truth definition for modal propositional logic
* If M is a model = {W, R, V}, then VM,w(Φ) (the truth value of Φ in w given M) is defined by the following clauses:

1. VM,w(p) = Vw(p), for all propositional letters p
2. VM,w(~Φ) = 1, if and only if VM,w(Φ) = 0
3. VM,w(Φ 🡪 Φ) = 1, if and only if VM,w(Φ) =0 or VM,w(Φ) =1.
4. VM,w(□Φ) = 1, if and only if for all w’ such that wRw’: VM,w(Φ) = 1

* Φ is necessarily true, is and only if for all worlds accessible from w, Φ is true.

1. VM,w(◊Φ) = 1, if and only if for at least one w’ such that wRw’: VM,w(Φ) = 1

* Φ is possibly true, is and only if at least one world which is accessible from w is true

Notes

* If there are 0 accessible worlds from w, ◊Φ is false because there are no worlds in which Φ can be true (at least once).
* BUT, □Φ cannot be inferred in the same way. Must be deduced FROM ◊Φ, using the Q.E. rule.
* ◊Φ = 0 <-> ~□~Φ = 0; therefore, □~Φ = 1.